Space-Time Bit Trellis Codes

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Abstract—Multiple antenna systems are a promising approach to increase the data rate of wireless communication systems. There exist two main transmission schemes. In BICM systems, modulated symbols are multiplexed on multiple transmission antennas while an outer error correction code ensures the desired quality of service for a given data rate. Space-time trellis codes use a trellis structure to determine the transmission symbols for different antennas. For both schemes, the decoding complexity grows exponentially with the number of antennas and thus the desired code rate.

In this paper, we propose the use of space-time bit trellis codes instead which are a concatenation of a bit convolutional code, modulator and spatial multiplexing without interleaving. We present a new near-ML decoding algorithm with an algorithmic complexity which scales only linearly with the number of antennas. This new algorithm can be applied to all schemes where trellis based channel coding is directly followed by modulation and spatial multiplexing. We show simulation results for 16-QAM up to 16x16 transmit/receive antennas with a data rate of 32 bits per channel use. The communications performance lies within 3dB of the outage capacity for a frame error rate of $10^{-7}$. This is to the best of our knowledge the best result reported so far.

I. INTRODUCTION

Multiple-antenna (MIMO) systems are a promising candidate for achieving high data rates in a rich-scattering wireless channel without increasing the bandwidth. In a spatial multiplexing MIMO system a symbol stream is demultiplexed to multiple transmit antennas while the receiver side collects superposed and noise disturbed samples by multiple receive antennas [1]. Transmission rates near MIMO capacity can be achieved by a serial concatenation of an outer channel code, interleaver, and modulator which can be seen as a classical bit interleaved coded modulation scheme (BICM). The outage capacity limit can be approached with BICM by iterative decoding [2][3]. High data rates of up to 12 bits per channel use are reported in literature [4]. However, the higher the data rate the larger the distance to the capacity bound. The main drawback of BICM systems is the high complexity of MIMO demodulation which is exponential in the number of antennas for optimal decoding. Furthermore the high latency of decoding due to iterations and interleaving is problematic as communications standards like e.g. WiFi [5] have very strict latency requirements.

The main techniques employed for MIMO channels to offer spatial diversity are space-time block codes (STBC) [6] and space-time trellis codes (STTC) [7]. Both offer full spatial diversity. Space-time block codes have no coding gain. Therefore, they are typically concatenated with an outer channel code separated by an interleaver ensuring locally uncorrelated bits for modulation. An iterative decoding is not possible, thus the (diversity) coding gain is limited. Space-time trellis codes combine coding and mapping in one step. Therefore, no additional channel code is needed. Communications performance is very good and can be within 3 dB of outage capacity. Decoding has a constant latency. However, its complexity increases exponentially with the data rate. Additionally, STTCs lack flexibility. For each combination of modulation and number of transmit antennas, a different code is needed. Feasible data rates typically found in literature are 2 or 3 bits per channel use [7][8]. For higher data rates, the computational complexity becomes too high.

In [9] a pragmatic encoding procedure for space-time codes was presented. Encoding is done by a standard bit convolutional code directly followed by a modulator and spatial multiplexing of the modulated symbols. Optionally an interleaver on transmission vector basis can be introduced. Therefore, one convolutional code can be used for different modulations and antenna constellations offering a high degree of flexibility like for BICM systems. However, in [9] the decoding complexity grows as well exponentially with the number of antennas. This paper presents a new decoding approach for trellis-based codes which are encoded on 'bit level' like e.g pragmatic space time coding or Ungerboeck type encoding.

We show that the 'bit level' encoding structure enables near-ML decoding for the joint processing of MIMO demodulation and convolutional code. Decoding is based on a trellis determined by the convolutional code. Each trellis step is associated with one modulated symbol instead of an entire transmission vector. Thus, the number of receive antennas only influences the length of the trellis but not the complexity of the single trellis steps. The overall complexity of the decoding algorithm depends only linearly on the number of receive/transmit antennas and is mainly determined by the number of states of the convolutional code. The near-ML decoding of space-time bit trellis codes has two major advantages:

- decoding latency is constant and small since the algorithm is not iterative,
- decoding complexity scales linearly with the number of antennas

For a 4x4 antenna system the communications performance of an iterative BICM system including a 64-state non-systematic convolutional (NSC) code is approached by 1 dB
by a corresponding STbitTC while the decoding complexity is much lower (Section IV). Furthermore we show communications performance results for a 16x16, 16-QAM system which reaches the outage capacity at FER=10^{-2} by 3 dB. To the best of our knowledge, this is the highest performance result reported so far for such a high data rate of 32 bits per channel use.

II. STATE OF THE ART SYSTEMS

Figure 1 shows typical MIMO transmitters for bit-interleaved coded modulation (BICM) and space-time trellis codes. For BICM, the source bits are encoded by an outer channel code of code rate $R$. The channel code can be a convolutional code, turbo-code or LDPC code respectively. The resulting codeword is interleaved and mapped onto complex symbols which are taken from a constellation of size $2^Q$. The symbols are multiplexed to $M_T$ transmit antennas and $M_T$ symbols per time step are transmitted over a MIMO channel. For details on the modeling of the transmission see Section III-A. The overall data rate is $\eta = R \cdot M_T \cdot Q$ which reflects the number of information bits per channel use. To obtain an error rate performance within 3 dB of the outage capacity, the decoding of BICM schemes is carried out in an iterative manner, where probabilistic messages are exchanged between MIMO demapper and channel decoder [2][3].

Space-time trellis codes combine coding and mapping in one step. The trellis code shown in Figure 1 b) implements an encoding scheme for 2 transmit antennas and four input symbols (4-QAM). The output is determined according to the branch labels depending on the current state. The initial state is $S_0$. The two branch labels of a transition are simultaneously transmitted by two antennas. The data rate $\eta$ depends on the number of different input symbols. In this example $\eta = 2$ bits per channel use. STTC can be maximum likelihood (ML) decoded by a vector Viterbi algorithm [7], for which the decoding complexity grows exponentially with the data rate.

Both systems achieve error rate performances within 3 dB of outage capacity. However, their major drawback is the very high decoding complexity which grows exponentially with the number of transmit antennas. Additionally, the decoding for BICM systems has a very high varying latency due to the iterations between MIMO demapper and channel decoder.

III. SPACE-TIME BIT TRELLIS CODE

BICM schemes are very flexible and can be decoded even for high data rates. However, a joint decoding of channel code and demodulation is not possible due to interleaving. Space-time trellis codes can be decoded optimally with a non-iterative Viterbi algorithm. However, high data rates are difficult to realize. The Space-time bit trellis codes (STbitTC) and its new decoding algorithm offer both, high data rates and a non-iterative decoding algorithm. Figure 2 shows 2 possible encoders for space-time bit trellis codes. Their composition are similar to a BICM system. Figure 2a employing a non-recursive convolutional (NSC) code of rate $R$. However, the interleaver is missing. The code rate $\eta = R \cdot M_T \cdot Q$ is identical to the BICM case. The information bits $u$ from the source are encoded by a convolutional encoder. The coded bits $X$ are directly mapped to constellation points (Gray mapping) without being interleaved.

A second possible encoding structure is to utilize trellis coded modulation with Ungerboeck symbol mapping followed by a direct multiplexing of the symbols to the transmit antennas (Figure 2b). The important point in both encoding schemes is that the code structure is preserved and can be used for joint decoding of MIMO detection and channel code. The encoding structure (Figure 2a) was already presented in [9] and denoted as pragmatic encoding. However, their ML decoding procedure is identical to that of classical STTC and thus exponential in the number of antennas. In this paper, we present a new decoding algorithm which has linear complexity with respect to the number of antennas. In the following paragraphs, the notation and the near-ML decoding procedure will be presented.

A. Generation and Transmission

The source generates a random infoword $u$ of length $K_c$ which is encoded by the channel encoder. The resulting codeword $X_1^N$ consists of $N_c$ bits which are grouped into $N$ subblocks $x_n$.

$$X_1^N = (x_1, x_2, \ldots, x_n, \ldots, x_N) \quad (1)$$

Each subblock $x_n$ consists of $Q$ coded bits.

$$x_n = (x_{1,n}, x_{2,n}, \ldots, x_{Q,n}, \ldots, x_{Q,n}) \quad (2)$$
The notation is taken from [10]. The codeword is not interleaved but each subblock $x_n$ is mapped directly to a complex symbol $s$ chosen from a $2^M$-ary QAM modulation scheme (Gray mapping). $M_T$ symbols are combined in one transmitted vector $s_t$.

\[
s_t = (s_{1,t}, s_{2,t}, \ldots, s_{M_T,t}) \quad (3)
\]

The whole modulated sequence is represented by

\[
S^T_t = (s_1, s_2, \ldots, s_t, \ldots, s_T) \quad (4)
\]

$T$ time slots are needed to transmit all symbols of one codeword. The transmission of one transmission vector $s_t$ in time step $t$ is modeled by multiplying it with the channel matrix $H_t$ and adding Gaussian noise $n_t$:

\[
y_t = H_t \cdot s_t + n_t \quad (5)
\]

With $H_t$ the channel matrix of dimension $M_T \times M_R$ and $n_t$ the noise vector of dimension $M_R$. The entries in $H_t$ are modeled as independent, complex, zero-mean, Gaussian random variables. Real and imaginary part are independent each with variance $\sigma^2 = N_0/2$. All received vectors $y_t$ are gathered in the matrix $Y^T_1$

\[
Y^T_1 = (y_1, y_2, \ldots, y_t, \ldots, y_T) \quad (6)
\]

with

\[
y_t = (y_{1,t}, y_{2,t}, \ldots, y_{M_R,t}) \quad (7)
\]

$H_t$ remains constant for $T$ time steps and thus one encoded block (static flat fading). It is assumed that $H_t$ is perfectly known by the decoder and that symmetric antenna constellations are employed with $M_T = M_R = M$. For ease of notation, the indices of $X, S, Y$ which denote their size and the time index of $H$ are dropped from now on.

### B. Detection Problem

The decoder is trying to find the most likely sequence $\hat{X}$ by solving the maximum likelihood criterion.

\[
\hat{X} = \text{argmax}_X P(Y|X, H) \quad (8)
\]

All sequences $X$ are equally likely. Therefore, the likelihood function is proportional to the negative squared Euclidean distance between received and transmitted symbols.

\[
\hat{X} = \text{argmin}_X ||Y - HS||^2 \quad (9)
\]

The solution can be interpreted as the path with minimum metric in a trellis diagram and can therefore be found by the Viterbi algorithm [7]. The Viterbi algorithm for STTC is shortly revised in the next section to explain its basic functionality and terms. Afterwards, the application of STbitTC on Viterbi decoding for near-ML decoding is presented.

### C. Revision of Viterbi Algorithm for STTC

The Viterbi algorithm finds the most likely sequence of state transitions (a path) through a trellis diagram of length $L$ with a finite number of states given the sequence of received symbols $y$. Decision metrics are the so called path metrics $\gamma$. They are calculated by summing up branch metrics $\gamma$ which depend on the received values $y_k$ and the assumed code bits $x_k$ belonging to trellis step $k$.

\[
\gamma(y_k | x_k) = \sum_{k=1}^L \gamma(y_k | x_k) \quad (10)
\]

The Viterbi Algorithm consists of two main steps. First, a forward recursion is carried out to determine the path metrics from the beginning to each possible state in the trellis. Therefore, a branch metric is assigned to each state transition. For each trellis step and state the local survivor is determined recursively by choosing the path with the smallest metric leading to this state:

\[
\mu_{k+1}^{(m')} = \min_m \left( \mu_k^{(m)} + \gamma_{k,k+1}^{(m,m')} \right) \quad (11)
\]

with $m'$ the trellis state in step $k+1$ and $m$ being a predecessor of $m'$ in step $k$. Decision bits indicating the local survivor for each state and each trellis step are stored in a survivor memory. In a proceeding traceback step the local survivors are read from the survivor memory backwards in the trellis to extract the most likely sequence. The traceback can be windowed since possible survivor paths merge after a certain traceback depth [11]. This is very important for low latency constraints of current standards, e.g. WiFi [5]. In classical STTC one trellis step corresponds to one time step $t$ or more explicitly the detection of one transmitted vector $s_t$. The trellis has a length of $T$ steps. The number of state transitions from each of the $2^A$ states is determined by the data rate $2^R$. The branch metrics are the euclidian distances of a received vector $y_t$ to an assumed transmitted vector $s_t$.

\[
\gamma(y_t | s_t) = ||y_t - HS_t||^2 \quad (12)
\]

### D. Near-ML Viterbi Decoding of STbitTC

Due to their binary trellis structure STbitTCs offer a high flexibility regarding modulation and antenna constellations. In the following sections we focus on the decoding for the encoder in Figure 2a. However, the described decoding method can be used for the encoder in Figure 2b as well. The binary trellis of a convolutional code has $K_c$ trellis steps (number of infobits). After MIMO transmission, information on single bits is not available. Goal is therefore to allocate one symbol $s_{j,t}$ to one trellis step. This can be achieved by merging [12] $RQ$ steps of the bit trellis at a time which correspond to a mapped symbol $s_{j,t}$. This new symbol trellis has the same number of states and a length of $N = NC/Q$ steps (see Figure 3). As the code structure has not been broken by an interleaver only

\[
1 \text{This symbol trellis is directly obtained when utilizing encoding scheme Figure 2b.}
\]
transitions have to be considered for each state instead of $2^Q$.

The derived symbol trellis structure and the number of transitions per state are independent of the antenna constellation. The only restriction applies to the choice of $R$ and $Q$ as their product has to be a natural number. Thus, various modulations and antenna constellations can be supported with the same convolutional code.

For an STbitTC in contrast to STTC, one trellis step corresponds to the detection of one symbol $s_{j,t}$ of one transmission vector $s_t$. Therefore the branch metrics in Equation (12) cannot be employed and a new branch metric is needed. In order to detect one symbol after the other, QR decomposition is applied to the channel matrix $H$ (compare sphere decoding algorithm [13]). This transformation results in an orthogonal matrix $Q$ and a lower-triangular matrix $R$ having the same dimensions as $H$. Using those matrices the Euclidian distance can be reformulated

$$\|y_t - H s_t\|^2 = \|\hat{y}_t - R s_t\|^2 + c$$  \hspace{1cm} (13)

with $\hat{y}_t = Q^T y_t$, and $c$ a real-valued constant which has no impact during path metric calculations so it can be neglected. The distance calculation can be further decomposed [13] [14].

$$\gamma_j = H s_j - H \sum_{j' \neq j} s_{j',t} = \sum_{j' \neq j} |s_{j',t} s_{j,t}^*(s_{j',t})|^2$$  \hspace{1cm} (14)

This allows us to define a new branch metric corresponding to one symbol per trellis step.

$$\gamma(y_{j,t} | s_t) = |c_{j,t}(s_{j,1-\ldots,j})|^2$$  \hspace{1cm} (15)

where $s_{j,1-\ldots,j}$ is a vector containing the symbols $(s_{1,t} \ldots s_{j,t})$. Thus, for the calculation of the current branch metric of symbol $s_{j,t}$, the $j-1$ previous symbols are needed. They can be obtained by a short traceback of the local survivors. The current symbol $s_{j,t}$ is determined by the state transition (Figure 3).

The choice of local survivors in each state induces a sub-optimality as the following branch metrics are not independent of previous decisions. Therefore, the algorithm performs only local survivors. The current symbol $s_{j,t}$ is applied to the channel matrix $H$ in order to detect one symbol after the other, QR decomposition is applied to the channel matrix $H$ (compare sphere decoding algorithm [13]). This transformation results in an orthogonal matrix $Q$ and a lower-triangular matrix $R$ having the same dimensions as $H$. Using those matrices the Euclidian distance can be reformulated [13] [14].

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The choice of local survivors in each state induces a sub-optimality as the following branch metrics are not independent of previous decisions. Therefore, the algorithm performs only near-ML.

### IV. COMPLEXITY ANALYSIS

In this section, the decoding complexity of STTC and STbitTC for the same data rate will be analyzed. We define the complexity of trellis based decoding as the product of trellis length, number of states, number of transitions per state and the complexity of calculating a branch metric.

For STTC decoding each trellis step corresponds to the detection of one transmission vector $s_t$ which relates to a trellis of length $T$. The number of states $S_A = 2^4$ is a design parameter. However, $S_A$ is usually a multiple of the modulation alphabet $2^Q$. From each state, $2^{R.M.Q}$ different transitions exist. The calculation of the branch metrics (Equation (12)) consists of $M$ Euclidean distances whereas only one Euclidean distance is needed for STbitTC. The branch metric calculation complexity for STTCs is therefore $M$ times higher.

The main difference of STbitTC is the granularity of the trellis structure. In contrast to STTC one trellis step corresponds to the detection of one symbol $s_{j,t}$ of one transmission vector $s_t$. Therefore, the trellis has a length of $N = M \cdot T$ steps. The number of states depends on the constraint length $B$ of the convolutional code, thus it has $S_B = 2^B$ states. It is independent of other system parameters. Each symbol consists of $Q$ coded bits. However, the code structure is known. Therefore, only the combinations of $R \cdot Q$ information bits have to be considered to determine the existing state transitions.

Table I summarizes the complexity for both, STTC and STbitTC. It also shows the overall complexity for both decoding algorithms. It can be seen that the decoding of STbitTC depends only linearly on the number of antennas $M$. This allows the transmission of very high data rates by maintaining a feasible decoding complexity.

<table>
<thead>
<tr>
<th></th>
<th>STTC</th>
<th>STbitTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>data rate</td>
<td>$R \cdot M \cdot Q$</td>
<td>$R \cdot M \cdot Q$</td>
</tr>
<tr>
<td>trellis length</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>number of states</td>
<td>$S_A$</td>
<td>$S_B$</td>
</tr>
<tr>
<td>transitions per state</td>
<td>$2^{R.M.Q}$</td>
<td>$2^{R.Q}$</td>
</tr>
<tr>
<td>branch metric compl.</td>
<td>$M$</td>
<td>1</td>
</tr>
</tbody>
</table>

**TABLE I**

Comparison of complexity of STTC and STbitTC.
V. SIMULATION RESULTS

For all simulations, codewords of length \( N_C = 1920 \) bits were used which were encoded by a channel code of rate \( R = 1/2 \). Furthermore, 16-QAM or 64-QAM was used and the channel matrix \( H \) was assumed to be constant within one transmitted codeword.

For STbitTCs different NSC codes were employed with generator polynomials \( g_0 = 133_o \), \( g_1 = 171_o \), \( g_0 = 561_o \), \( g_1 = 753_o \), \( g_0 = 10627_o \), \( g_1 = 16765_o \) comprising 64-states, 256-states, 1024-states, and 4096-states.

Tailbiting was used as encoding scheme to keep the desired code rate of \( R = 1/2 \) except for the 16x16 antenna setup where tail bits were adopted resulting in a very small rate degradation to \( R = 0.494 \) for both presented STbitTC schemes. This was mandatory since 30 transmission vectors \( s_t \) are too short for the mandatory acquisition to initialize tailbiting states. We concentrate on the encoding according to Figure 2a. Results for TCM (Figure 2b) will be shown in a follow-up paper. Since all simulations are focused on high data rates, no performance results for STTC systems are presented due to the lack of codes and their decoding complexity.

Figure 4 shows the communications performance of STbitTCs (Figure 2a) compared to a state-of-the-art BICM system. The closed loop performance curve of the BICM scheme with 64-state NSC code (same generators) was obtained with 5 decoder iterations between MIMO demapper [3] and channel decoder. More iterations do not yield a significant performance increase. The newly presented STbitTC with 64 states employing the approximated ML decoding algorithm presented in Section III-D approaches the iterative BICM decoding by 1 dB at FER=10^{-2}. For NSC codes with more states (256 and 1024) the communications performance can be further improved.

Figure 5 shows the communications performance for 4x4 and 64-QAM modulation with a data rate of \( \eta = 12 \) bits per channel use. The BICM reference performance is a WiMAX LDPC code of block length of 1920 bits and code rate \( R=1/2 \). 5 inner LDPC iterations and 8 feedback iterations between a soft-in soft-out demodulator and LDPC decoder are performed. The performance of an STbitTC with 64-state NSC code is even better than the iterative BICM scheme. STbitTC decoding with a 4096-state NSC code approaches the capacity bound by 2 dB.

Figure 6 and Figure 7 show the communications performance for 8x8 and 16x16 antennas with a data rate of \( \eta = 16 \) and \( \eta = 32 \) bits per channel use. No reference simulations are shown for these simulations, since the complexity of the iterative BICM scheme was too high to achieve a comparable performance. Especially the 16x16 antenna system with a 4096-state NSC code shows very good results with a performance approaching the outage capacity by 3 dB at FER=10^{-2}.

Note that STbitTC decoding is non-iterative. Thus, it offers a possibility for continuous decoding (streaming applications) which is of great interest for standards with stringent latency requirements \(^2\), e.g. WiFi standard [5].

VI. CONCLUSIONS

Space-time trellis codes combine the advantage of diversity and coding gain and provide an excellent communications performance. Bit interleaved coded modulation is a flexible approach providing excellent performance for iterative demodulation/decoding. However, the decoding complexity of both methods grows exponentially with the number of antennas. We presented a simple and flexible encoding scheme with a new ML approximating decoding method which scales linearly with the number of antennas. The communications performance of this new space-time bit trellis coding (STbitTC) is comparable to state of the art systems for lower data rates.

\(^2\) For low latency applications the trellis has to be terminated by tail bits rather than tailbiting.
and outstanding for very high data rates as shown for 16x16 antennas system with 32 bits per channel use.

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REFERENCES


