Turbo-Decoder Quantization for UMTS

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Abstract— **The use of Turbo-Codes is proposed for high-rate data services in third generation wireless communication systems. Bit-true models are mandatory for hardware and software implementations. In this paper we present to the best of our knowledge the first investigation of a combined bit-width optimization of input data and internal data for an 8-state Turbo-decoder based on parameters relevant for UMTS. Simulation results for AWGN and Rayleigh-fading channels show that performance degradation can be held below 0.11 dB using a 4-bit input data quantization.**

Keywords—**Quantization, Turbo-Codes, Mobile communication**

I. Introduction

TURBO-CODES are part of the standardization process for third generation wireless communication systems (UMTS). Thus, efficient Turbo-decoder implementations with emphasis on low cost and low energy consumption are of emerging importance. Digital signal processing algorithms like the MAP algorithm used in a Turbo-decoder are usually specified in the floating-point domain. Fixed-point number representation is mandatory for most target architectures, thus transformation from floating to fixed-point is a necessary step towards an actual implementation [1]. The resulting bit-true models of the Turbo-decoder may differ for software, hardware and mixed hardware/software target architectures. Primary goal for a software implementation is to find a fixed-point model that corresponds to the given bit-width of the DSP. Further bit-width minimization can reduce the switching activity and has thus influence on the power consumption. Primary goal for a dedicated hardware implementation is to choose all bit-widths as small as possible, resulting in a reduction of area and energy consumption. Hence, an optimized quantization has a large impact on the implementation cost.

Only few papers have been published on Turbo-decoder quantization. They address input data quantization, assuming infinite accuracy of the internal values [2] or presume a unified quantization of all signals [3], [4]. An 8-state Turbo-Code is considered in [5], however not under fading-channel conditions nor with 3GPP compliant interleavers. In [6] we have for the first time discussed a systematic approach towards an internal quantization scheme of a 4-state Turbo-decoder with infinite accuracy of the input data. In this paper we present a combined optimization of input data quantization and internal quantization for a 3GPP compliant 8-state Turbo-decoder.

The remainder of this paper is structured as follows: A short review of the system model for the Turbo-decoder is given in Section II. Section III addresses issues of the input data quantization and explains consequences on the quantization model of the MAP algorithm. In Section IV simulation results are presented. Section V concludes this paper.

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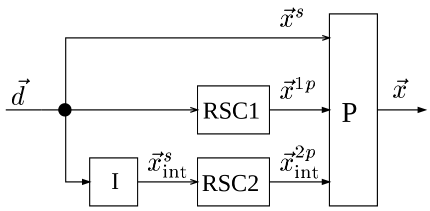

Fig. 1. Turbo-Code Encoder

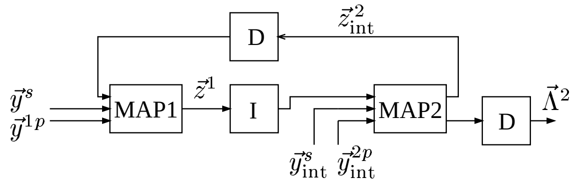

Fig. 2. Turbo-decoder

II. System Model

In the following we assume a typical Turbo-encoder with a structure proposed in [7] (see Fig. 1). It consists of two identical constituent code encoders (RSC1, RSC2), each generating a recursive systematic convolutional code of constraint length $K = 4$ using generator polynoms 13/15. This configuration complies with the 3GPP proposal for a code with $M = 2^{K-1} = 8$ states [8]. The two encoders are separated by an interleaver (I), which also complies with the 3GPP proposal. The parity information ($\vec{x}^{1p}$, $\vec{x}^{2p}_{int}$) and the systematic information data sequences ($\vec{x}^s$) are concatenated in parallel, leading to three output bits for each bit of the input data sequence $\vec{d} = (d_1, \ldots, d_N)$, where $N = 600$ is the block size. In a puncturing and rate matching unit (P) the code rate is adapted. We selected a puncturing scheme which omits every second bit of the parity information, leading to a transmitted data sequence $\vec{x} = (x^s_1, x^{1p}_1, x^s_2, x^{2p}_{int,2}, \ldots, x^s_{N-1}, x^{1p}_{N-1}, x^s_N, x^{2p}_{int,N})$ and a total code rate of $R = 0.5$. No tailing scheme is implemented.

For decoding this Turbo-Code we use the conventional symmetric decoder structure (see Fig. 2), consisting of two maximum a posteriori (MAP1, MAP2) decoders [9], one interleaver (I) and two deinterleavers (D). Each of the MAP decoders corresponds to one constituent code. The reader is referred to [10] for a more detailed discussion of the MAP algorithm. A short review of the Turbo-decoder implementation, which is relevant in the scope of this paper is given in [6].

III. Input Quantization

We have demonstrated in [6] that an optimized quantization scheme of the internal values does not degrade the bit-error performance. In this paper, optimized quantization schemes for input data *and* internal values are considered. The notation (q, f)

z_k	< -1	-1.0	-0.5	0.0	0.5	1.0	> 1.0
$\ln(1+e^{z_k})$	0	0.5	0.5	0.5	1.0	1.5	z_k

TABLE I
LOOKUP-TABLE FOR A QUANTIZATION SCHEME I:(4, 1)

$\lvert\delta_2-\delta_1\rvert$	0.00	0.25	0.50	0.75	1.00
$\ln(1+e^{-\lvert\delta_2-\delta_1\rvert})$	0.75	0.50	0.50	0.50	0.25
$\lvert\delta_2-\delta_1\rvert$	1.25	1.50	1.75	2.00	> 2.00
$\ln(1+e^{-\lvert\delta_2-\delta_1\rvert})$	0.25	0.25	0.25	0.25	0

TABLE II
LOOKUP-TABLE FOR CORRECTION TERM WITH GRANULARITY OF $g = 2^{-2}$

is used to describe the fixed-point representation of a signal: q represents the total bit-width of the fixed-point number and f the bit-width of the fractional part. Thus, $q - f$ bits represent the dynamic-range and $g = 2^{-f}$ the granularity. Input data is denoted as I:(q, f) in the quantized case and as I:float in the case of infinite accuracy.

For BPSK and an AWGN channel, the received values are distributed with a Gaussian distribution around the transmitted symbols {-1,1}. More than 99% of the occurring values are covered by limiting the dynamic-range of the received channel values to [-4,4). This dynamic-range is reasonable also for a Rayleigh-fading channel and can be represented by 3 bits in a uniform quantization. Our simulations confirm for both channel models that increasing the number of bits for the dynamic-range delivers no gain [5]. Thus, the total bit-width q of the input data is determined by the granularity bits f: $q = 3 + f$. In Section IV we present simulation results for quantization schemes of the input data with $f = \{0,\ 1,\ 2\}$, denoted as I:$(3, 0)$, I:$(4, 1)$, and I:$(5, 2)$.

The transition metrics $\overline{\gamma}_i$ of the MAP algorithm are calculated as weighted sums of the channel input data and the a priori information ([6], eqns.(4),(5))

$$\overline{\gamma}_i\left[(y_k^s, y_k^p), S_{k-1}, S_k\right] = \frac{E_s}{N_0}(y_k^s - x_k^s(i))^2 + \frac{E_s}{N_0}(y_k^p - x_k^p(i, S_k, S_{k-1}))^2 + \ln \Pr\{S_k|S_{k-1}\} \quad (1)$$

($i = 0, 1$), where $\ln \Pr\{S_k|S_{k-1}\}$ comprises the a priori information:

$$\ln \Pr\{S_k|S_{k-1}\} = \begin{cases} z_k - \max{}^*(0, z_k), & q(d_k = 1|S_k, S_{k-1}) = 1 \\ -\max{}^*(0, z_k), & q(d_k = 0|S_k, S_{k-1}) = 1 \end{cases} \quad (2)$$

and

$$\max{}^*(\delta_1, \delta_2) = \max(\delta_1, \delta_2) + \ln(1 + e^{-|\delta_2-\delta_1|}). \quad (3)$$

The granularity g of the a priori information z_k is adapted to the granularity of the input data. In software implementations this reduces the complexity, because costly shift operations are avoided. The extrinsic information is coupled in accordance with eqn. (2). The function $\max^*(0,\ z_k) = \ln(1 + e^{z_k})$ can be replaced with a small lookup-table without degradation of the bit-error performance, because the quantization scheme of z_k is known. The lookup-table for I:$(4, 1)$-quantized input data and $(6, 1)$-quantized z_k is given in Table I.

IV. SIMULATION RESULTS

Under the assumption of I:float, simulation results for the 8-state Turbo-decoder show the same behavior as for the 4-state Turbo-decoder [6]: No degradation for the $(9, 4)$-quantized Log-MAP model is observed in comparison to the reference floating-point model. Reducing the bit-width of the input data to I:$(5, 2)$, almost the same performance is maintained. For example, at an operating point with $\text{BER} = 10^{-5}$ degradations of about 0.04 dB for an AWGN channel and even improvements of up to 0.013 dB for a Rayleigh-fading channel are observed. With this I:$(5, 2)$ quantization scheme the internal quantization scheme can be reduced to $(7, 2)$ without additional degradation. Implementing the correction term of the Jacobian logarithm function (eqn. (3)) as a small lookup-table with input and output granularity of $g = 2^{-2}$ (see Table II) comes at no degradation of the SNR, but reduces the implementation complexity significantly. A further reduction of the input data bit-width to I:$(4, 1)$ or I:$(3, 0)$ leads to degradations of the SNR, which are shown in Table III for $\text{BER} = 10^{-5}$, AWGN/Rayleigh-fading channels and 5/10 decoding iterations. The corresponding bit-error curves for the 3GPP compliant 8-state Turbo-decoder are depicted in Figures 3, 4, 5, and 6.

	AWGN		Rayleigh-fading	
	5 Iter.	10 Iter.	5 Iter.	10 Iter.
Log-MAP:$(9, 4)$, I:float	-0.016	0.00	0.013	0.00
Log-MAP:$(7, 2)$, I:$(5, 2)$	0.022	0.044	-0.013	0.00
Log-MAP:$(7, 2)$, I:$(4, 1)$	0.070	0.11	0.076	0.095
Log-MAP:$(7, 2)$, I:$(3, 0)$	0.16	0.17	0.22	0.22

TABLE III
DEGRADATION OF SNR IN [dB] OF DIFFERENT QUANTIZATION SCHEMES COMPARED TO THE REFERENCE FLOATING-POINT MODEL AT $\text{BER} = 10^{-5}$, 8-STATE TURBO-CODE

Simulation results have shown that for I:$(4, 1)$ and I:$(3, 0)$ the quantization scheme of the algorithm cannot be further reduced, according to the granularity of the input data, to $(6, 1)$ or $(5, 0)$, respectively; the $(7, 2)$ quantization scheme still performs best. The increase of granularity within the algorithm is caused by the correction term of Table II. Thus, in order to counteract the degradation caused by the input data quantization, algorithmic accuracy can be traded to some extent for the accuracy of the input data. This is impossible in the case of a Max-Log-MAP implementation, because of the approximation $\max^*(\delta_1, \delta_2) \approx \max(\delta_1, \delta_2)$. The granularity of the algorithm always corresponds to the granularity of the input data; the internal bit-width is therefore further reduced to $(6, 1)$ for I:$(4, 1)$-quantized input data.

V. CONCLUSIONS

Several quantization schemes for the input data bits have been discussed in combination with internal quantization schemes of an 8-state Turbo-decoder, based on parameters relevant for

UMTS. Simulation results of this combined bit-width optimization of input data and internal data show that a 4-bit quantization of the channel input data in combination with a 7-bit quantization for the Log-MAP algorithm is a reasonable compromise between implementation complexity and degradation of decoding performance, which is only about 0.1 dB for both AWGN and Rayleigh-fading channels.

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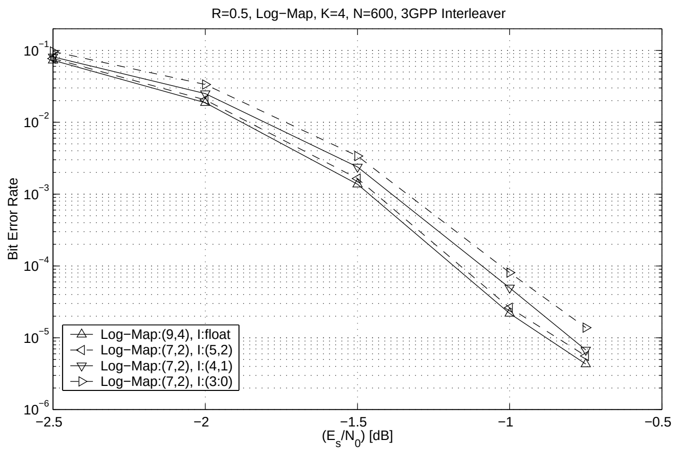

Fig. 3. Bit-error performance of a Turbo-decoder, AWGN channel, 5 Iterations

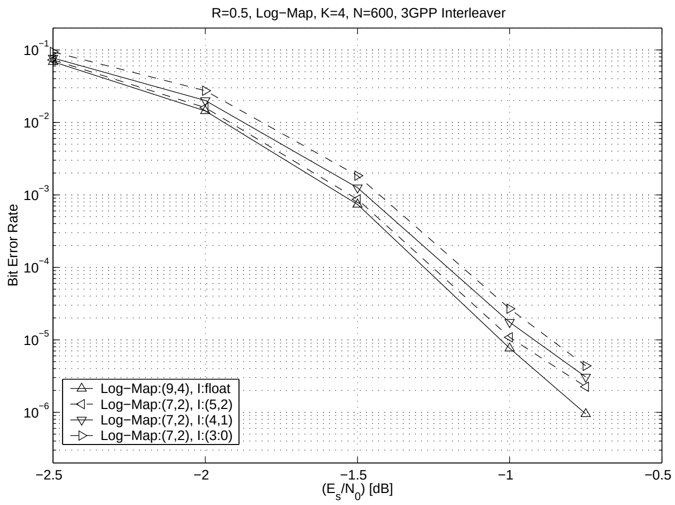

Fig. 4. Bit-error performance of a Turbo-decoder, AWGN channel, 10 Iterations

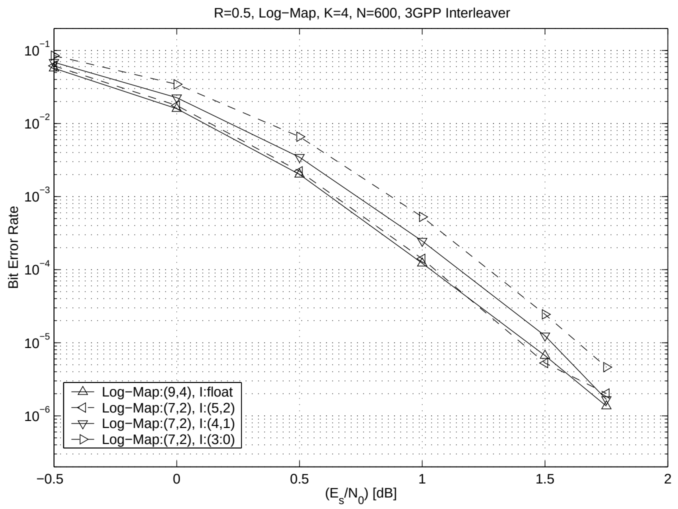

Fig. 5. Bit-error performance of a Turbo-decoder, Rayleigh-fading channel, 5 Iterations

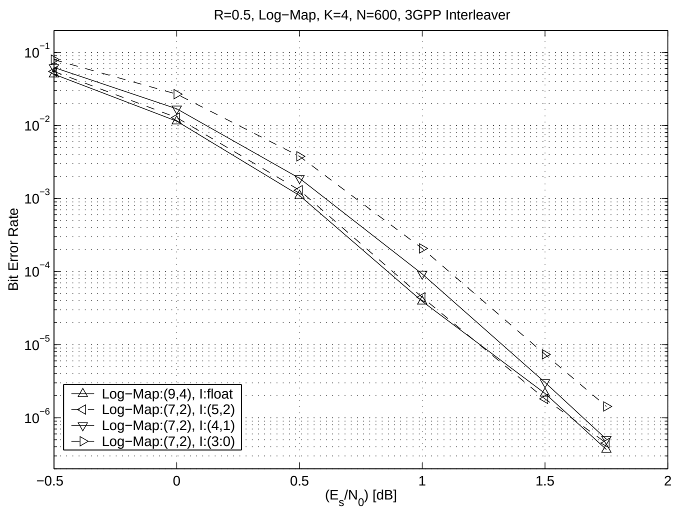

Fig. 6. Bit-error performance of a Turbo-decoder, Rayleigh-fading channel, 10 Iterations

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